5B.1

A GEOMETRIC APPROACH TO CHARACTERIZING AND INTERPRETING DUAL-POLARIZATION RADAR METEOROLOGICAL OBSERVATIONS

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Geometric interpretations provide a powerful way of envisioning physical phenomena and effects. The Poincaré sphere provides such a way of characterizing the polarization state of electromagnetic radiation, but has been little used in radar meteorology. It is in fact quite useful in understanding and interpreting dual-polarization meteorological measurements.

The polarization state of an electromagnetic signal is characterized by four quantities. The quantities can be expressed in a number of ways; three are required to describe the polarized part of the signal and one is needed to describe the unpolarized part. The Poincaré representation describes the polarized part of a signal in terms of its location in the three dimensional space of the Stokes parameters Q,U,V (Figure 1). For a given polarized power I_p, the polarization state lies on the surface of a sphere whose radius is $\mathrm{I}_p = \sqrt{\mathrm{Q}^2 + \mathrm{U}^2 + \mathrm{V}^2}$. Linear polarization states of different orientation angles lie around the equator of the sphere, while left- and right-hand circular polarizations are at the north and south poles.

Each Stokes parameter corresponds to the *difference* of the powers in orthogonal sets of polarizations. Denoting the powers of the H and V components of a signal by W_H and W_V, the Stokes parameter $\mathrm{Q} = W_H - W_V$. Similarly, $\mathrm{U} = W_+ - W_-$ and $\mathrm{V} = W_L - W_R$, where W_+,W_- are the signal powers in a $\pm 45°$ linear basis and W_L,W_R are the powers in a left- and right-hand circular basis. A horizontally polarized signal has equal amounts of $\pm 45°$ and L,R power, so that $\mathrm{U} = \mathrm{V} = 0$. In addition, it has no vertically polarized power. Thus, the H polarization state lies on the +Q axis. Similarly, a pure vertical polarization lies on the −Q axis, left- and right-circular polarizations lie on the ±V axes, etc. A general elliptical polarization lies at some location $P = (\mathrm{Q,U,V})$ between the equator and the poles of the sphere.

The fourth polarization parameter is the unpolarized power I_u or, equivalently, the total power of the signal, $\mathrm{I} = \mathrm{I}_p + \mathrm{I}_u$. The *degree of polarization* is the ratio of the polarized power to the total power, $p = \mathrm{I}_p/\mathrm{I} \leq 1$. By considering the total power to be normalized to unity, the radius of the Poincaré sphere is the degree of polarization p.

For meteorological and other applications, the location of the polarization state on the Poincaré sphere is best described in spherical rather than cartesian coordinates. The spherical angles are defined in one of two ways depending on which axis is considered to be the polar axis. When the polar axis is the circular polarization axis V, the azimuthal angle is defined to be 2τ and the polar angle is expressed in terms of its complement 2δ up from the equator (Figure 3). τ corresponds to the orientation angle of the polarization ellipse and δ measures the ellipticity of the polarization. The spherical coordinates (δ,τ,p) completely specify the polarization state when the total power I is normalized to unity.

Figure 2 shows how backscatter from horizontally oriented particles such as rain changes the polarization state. The differential reflectivity Z_{DR} of the particles increases the horizontal component of the signal relative to the vertical component and causes the polarization state to be displaced along a great circle toward the H polarization point. Differential attenuation by rain (DA) does the opposite and causes the polarization state to move towards the V polarization point. Differential phase effects cause the polarization state to rotate around the sphere in a plane perpendicular to the Q axis. Propagation differential phase ϕ_{dp} rotates the polarization state in a clockwise direction around the sphere; differential phase upon backscatter, δ_ℓ, does the opposite but usually has a negative sign for rain and therefore adds to ϕ_{dp}. Variability in the relative shapes of the drops introduces an unpolarized component in

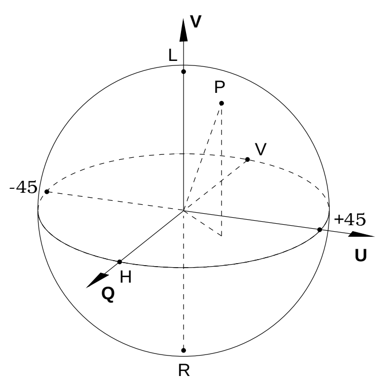

Figure 1. The Poincaré sphere representation of the polarization state.

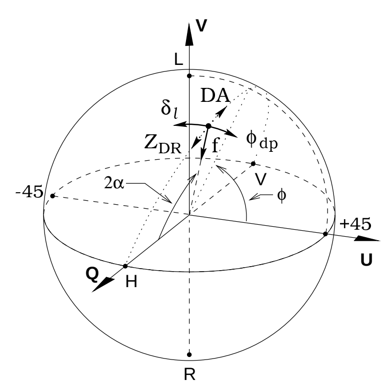

Figure 2. The effect of scattering by horizontally oriented particles.

the backscattered return and decreases the degree of polarization p. When the incident polarization state has equal or nearly equal H and V powers, each of the changes are in orthogonal directions on the Poincaré sphere.

The above indicates how the scattering parameters of rain can be determined for a general incident polarization. The changes have azimuthal symmetry about the Q or H–V axis of the Poincaré sphere and are best described in a spherical coordinate system having Q as its polar axis (Figure 2). The polar angle is 2α and measures the relative amount of H and V power in the signal. The azimuthal angle is ϕ and corresponds to the phase difference between the H and V components. The change in α is greatest when the H and V powers are equal or nearly equal; this is the situation for circular or $\pm 45°$ linear transmissions. The depolarization is independent of whether circular or $45°$ linear polarization is transmitted; this affects only the starting value of ϕ. H and V incident polarizations, being on the axis of symmetry, by themselves experience no depolarization and must be transmitted in combination to determine the scattering parameters.

Figure 3 shows how randomly oriented particles alter the polarization state. The changes are azimuthally symmetric about the V or circular-polarization axis of the sphere and are governed by a single quantity, termed the sphericity parameter g. Circular and linear incident polarizations are unchanged by the scatterers, except some of the polarized power is converted to unpolarized power (arrows a and b). The resulting change in p is at least a factor of two greater for circular than for linear polarization. Elliptical polarizations are partially unpolarized and are also made more linear by the scattering (arrows c). The changes are best described in the (δ,τ,p) spherical coordinate system.

Figure 4 shows the polarization changes produced by particles aligned at an angle τ relative to horizontal. The changes are the same as for horizontally oriented particles, but occur about a Q$'$ axis rotated an amount 2τ from the Q axis. Circular polarization measurements are therefore able to determine particle alignment directions by virtue of the direction of ϕ_{dp} and/or $Z_{\rm DR}$ changes on the Poincaré sphere.

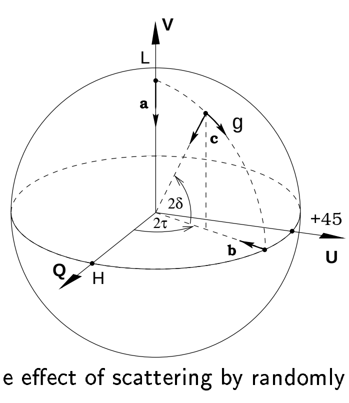

Figure 3. The effect of scattering by randomly oriented particles.

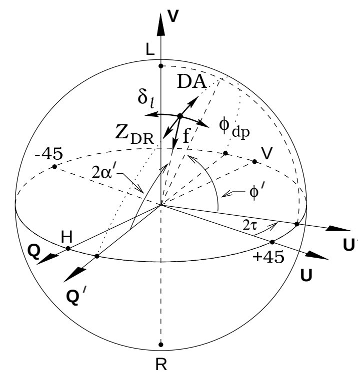

Figure 4. The effect of scattering by aligned particles oriented at angle τ relative to horizontal.

A quantitative description of the above changes is given by Scott (1999), Scott et al. (2001), and Krehbiel and Scott (2001). (See also companion paper 5B.2 of this conference.) The basic measurements used to determine the polarization state are the covariances W_1 and W_2 in two orthogonal polarizations and the magnitude and phase of the cross-covariance W between the two polarizations. The measurements have traditionally been made using alternate H and V transmissions but are optimally obtained from simultaneous transmissions, such as circular and/or slant $45°$ linear polarizations. The backscattered returns are best measured in H and V receiving channels, which tends to equalize the signal-to-noise ratios in each channel. For this case, $W_1 = W_H$, $W_2 = W_V$, and $W = \rho_{HV} e^{j\phi_{HV}}$. The phase difference ϕ_{HV} is identical to ϕ, the azimuthal angle of Figure 2. Also, for equal or nearly equal H and V powers, the correlation magnitude ρ_{HV} is the same as the degree of polarization p and therefore directly measures changes in p. For randomly oriented scatterers, p is decreased by a factor of $g/(2-g)$ when the incident polarization is circular and by $1/(2-g)$ for linear incident polarization. As discussed in the above references and in paper 5B.2, g is given by

$$g \quad = \quad \frac{4{\rm Re}\{\langle S_{xx} S_{yy}^* \rangle\}}{\langle |S_{xx} + S_{yy}|^2 \rangle} \qquad (1)$$

and measures the departure of the particles from sphericity. For small depolarizations, it is readily shown that the change in p is twice as great for incident circular polarization than for linear polarization.

The geometric interpretations provide a simple way of understanding dual-polarization observations and a quantitative framework for analyzing measurements. The following observations can be made from a qualitative standpoint (Scott et al., 2001):

1. H and V polarizations do not need to be transmitted separately; the parameters of both horizontal and randomly oriented particles can be determined from simultaneous H and V transmissions, in the form of circular, slant $45°$ linear or elliptical polarization. Simultaneous transmissions are optimal in that the measurements

are not affected by pulse-to-pulse Doppler effects (e.g., Doviak et al., 2000; paper P1.8 of this conference). Also, the measurements do not require a polarization switch.

2. The symmetries of the scattering effects show that circular and slant linear polarizations are equally affected by horizontally oriented particles such as rain, but are differently affected by randomly oriented particles such as hail. If only a single polarization is transmitted, the optimal choice would be circular, which is twice as sensitive as linear to the effects of randomly oriented particles.

3. The fact that circular and linear polarizations respond differently to randomly oriented particles but not to horizontally oriented particles provides a basis for objectively separating out the contributions by the two classes of scatterers. This would be done by transmitting, say, LHC and $+45°$ linear polarization on alternate pulses. This kind of polarization diverse measurement become possible by not having to alternate between H and V transmissions.

4. Alternating between $+45°$ and $-45°$ slant linear transmissions is not particularly useful, as the depolarization produced by horizontal and randomly oriented scatterers is the same in both cases and provides no additional information on the particles.

5. The Poincaré representation shows geometrically how circular polarization transmissions can be used to determine particle alignment directions. Referring to Figure 4, if the aligned particles primarily cause propagation differential phase shift (as electrically aligned ice crystals appear to do), the orientation angle can be determined from the direction of the ϕ_{dp} change using a single transmitted polarization (e.g., Krehbiel et al., 1996). If the scatterers also have measurable differential reflectivity, the alignment direction can be unambiguously determined by alternating between LHC and RHC transmissions. Again referring to Figure 4, the incident polarization state alternates between the top and bottom of the Poincaré sphere. $Z_{\rm DR}$ effects remain in the same direction (i.e., toward the $+{\rm Q}'$ axis), while differential phase effects are in opposite directions and therefore can be separated out.

6. The linear depolarization ratio LDR measures, say, W_V/W_H when H polarization is transmitted. LDR detects randomly oriented particles by virtue of their effect on the degree of polarization, and particle canting through the effect of $Z_{\rm DR}$ and ϕ_{dp} of the canted particles on an incident H polarization[1]. Randomly oriented particles are better detected by means of the correlation coefficient ρ_{HV} of simultaneous transmissions, which is a coherent rather than incoherently measured quantity. If canting information is not important or is not needed, LDR does not need to be measured. For randomly oriented particles, LDR and ρ_{HV} are related by

$$\rho_{HV} = \frac{1 - LDR}{1 + LDR}\,, \qquad (2)$$

when ρ_{HV} is measured from simultaneous transmissions (Scott et al., 2001).

In summary, the geometric approach enables one to visualize how different types of scatterers affect the polarization state of the radar signal, and how different types of polarizations can be used to determine the various scattering parameters. One result of this is a unification of the linear- and circular-polarization approaches to radar meteorological measurements. Another result is that it shows how different types of polarizations can be used to separate out the contributions of different classes of scatterers.

Although not addressed in this paper, the Poincaré approach also provides a valuable framework for formulating the scattering problem analytically and in a fully general way (see for example paper 5B.2). Particularly important in this regard is the fact that variability in the degree of polarization is simply and straightforwardly taken into account. The formulations provide a clear understanding of the transformations between different polarization bases and between different ways of representing the polarization state, that are useful both in understanding the depolarization effects and in practical issues such as feed alignment requirements and the calibration of dual-polarization systems.

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[1] Referring to Figure 4, if the incident polarization is H, $Z_{\rm DR}$ of canted particles causes the polarization state to move from the H polarization point toward the $+{\rm Q}'$ axis and therefore introduces a vertical polarization component and a non-zero LDR value. ϕ_{dp} of the canted particles does the same by causing the polarization state to rotate in a circular direction around the ${\rm Q}'$ axis.